

$n = p_1^{a_1} p_2^{a_2} \dots p_t^{a_t}$ with p_1, \dots, p_t distinct primes and $p_1 > \dots > p_t$.

- $p_1 \geq 10\,000\,019$ from Jenkins [5], improving on Hagis and Cohen [10]
- $p_2 \geq 10\,007$ from Iannucci [9], improving on Pomerance [19]
- $p_3 \geq 101$ from Iannucci [7]
- $p_{t-i} < 2^{2^i} (t-i)$ for $1 \leq i \leq 5$ from Kishore [16]
- $p_t < \frac{2}{3}t + 2$ from Perisastri [24]
- $p_k^{a_k} > 10^{20}$ for some k from Cohen [13], improving on Muskat [23]
- $n > 10^{300}$ from Brent, et. al. [11], improving on Brent and Cohen [12]
- $n \equiv 1, 9, 13, 25 \pmod{36}$ from Touchard [25]
- $n < 2^{4^t}$ from Nielsen [6], improving on Cook [8]
- $\Omega(n) = a_1 + a_2 + \dots + a_t \geq 63$ from Hare [1][2], improving on Iannucci and Sorli [4]
- $\omega(n) = t \geq 8$ from Hagis [18]¹, improving on Pomerance [20]
- $\omega(n) = t \geq 11$ if $p_t > 3$ from both Hagis [14] and Kishore [15]

Suryanarayana and Hagis [22] showed that in all cases $0.596 < \sum_{p|n} \frac{1}{p} < 0.694$. Their paper has more precise bounds based on the divisibility of n by 3 and 5. Cohen [17] gives stricter ranges for the same sum.

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¹E. Z. Chein's Ph.D thesis came to this conclusion one year earlier, but Hagis and most of the mathematical community were unaware of the result until after Hagis' proof.

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